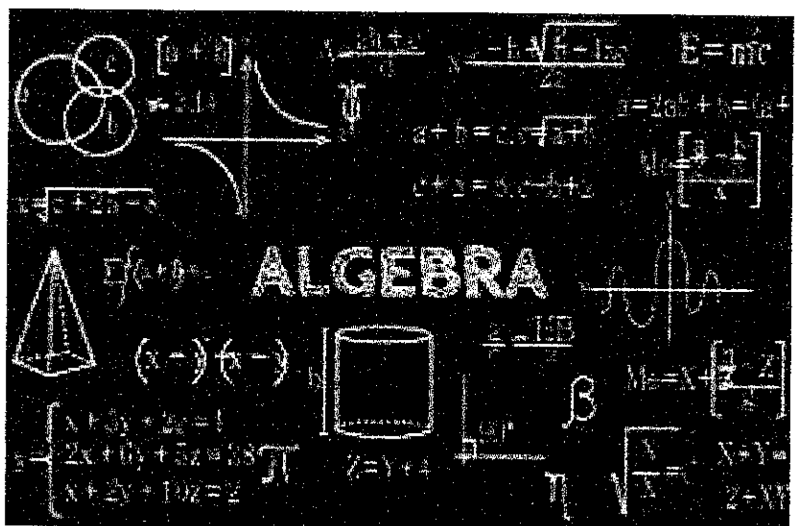
Algebra Summer Packet 2020-2021 School year



no WORK = no credit
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Name:____________________

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Greatest Common Factor

Factors that are shared by two or more numbers are called **common factors.** The greatest of the common factors is called the **greatest common factor** (GCF). There are several different ways to find the GCF of two or more numbers.

Example 1 **Find the greatest common factor (GCF) of 56 and 104.**

Method 1 List the factors of each number. Then circle the common factors.

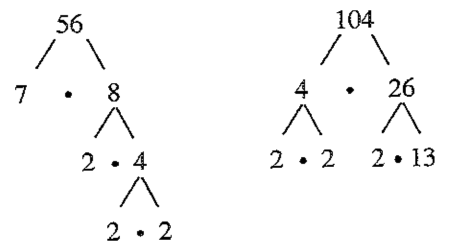
Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56

Factors of 104: 1, 2, 4, 8, 13, 26, 52, 104

The common factors are 1, 2, 4, and 8. The greatest of these common factors is 8.

▶ So, the GCF of 56 and 104 is 8.

Method 2 Make a factor tree for each number.



Write the prime factorization of each number. Then circle the common prime factors. The GCF is the product of the common prime factors.

$$56 = 2 \cdot 2 \cdot 2 \cdot 7$$
$$104 = 2 \cdot 2 \cdot 2 \cdot 13$$

▶ So, the GCF of 56 and 104 is $2 \cdot 2 \cdot 2 = 8$.

Practice

Check your answers at BigIdeasMath.com.

Find the GCF of the numbers using the two methods shown above.

1. 30, 45 **2.** 12, 54 **3.** 16, 96 **4.** 42, 98

5. 27, 66 **6.** 50, 160 **7.** 21, 70 **8.** 76, 95

9. 60, 84 **10.** 60, 120, 210 **11.** 44. 64, 100 **12.** 15, 28, 70

13. Write a set of two numbers that have a GCF of 20. Explain how you found your answer.

14. Write a set of three numbers that have a GCF of 25. Explain how you found your answer.

15. **BOUQUETS** A florist is making identical bouquets using 90 white roses, 60 red roses, and 45 pink roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?

16. **FABRIC** You have two pieces of fabric. One piece is 6 feet wide and the other piece is 7.5 feet wide. You want to cut both pieces into strips of equal width that are as wide as possible. How wide should you cut the strips of fabric?

Sets of Numbers in the Real Number System

Reals

A real number is either a rational number or an irrational number.

$4, -7, 0, \frac{2}{3}, \sqrt{11}$

Rationals

A rational number is any number that can be put in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

$3, \frac{12}{6}, -\frac{5}{2}, 4\frac{1}{7}, \frac{8}{13}$

Irrationals

An irrational number is a nonrepeating, nonterminating decimal.

$\sqrt{2}, \pi, -\sqrt[3]{7}, 0.121231234\ldots, \sqrt{13}$

Non-Integer Fractions

A non-integer fraction is a fraction whose numerator is *not* a multiple of the denominator.

$\frac{1}{2}, -\frac{5}{4}, 8\frac{3}{7}$

Integers

The integers consist of the natural numbers, 0, and the opposites of the natural numbers.

$\ldots -2, -1, 0, 1, 2, 3, \ldots$

$\ldots -5, -4, -3, -2, -1$

Whole Numbers

The whole numbers consist of the natural numbers and 0.

$0, 1, 2, 3, 4, \ldots$

0

Natural Numbers

The natural numbers are also referred to as the counting numbers.

$1, 2, 3, 4, \ldots$

The Number System

Identify the sets to which each of the following numbers belongs by marking an "X" in the appropriate boxes.

	Number	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
1.	$-\sqrt{17}$						
2.	-2						
3.	$-\frac{9}{37}$						
4.	0						
5.	-6.06						
6.	$4.5\overline{6}$						
7.	3.050050005...						
8.	18						
9.	$\frac{-43}{0}$						
10.	π						
11.	$.\overline{634}$						
12.	$\sqrt{225}$						
13.	$.634$						
14.	$\sqrt{\frac{4}{49}}$						
15.	$-\sqrt{64}$						

	Number	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
16.	$\sqrt{13}$						
17.	-5						
18.	$\frac{2}{3}$						
19.	-0.083						
20.	27						
21.	$2.6\overline{47}$						
22.	$3.0\overline{505}$						
23.	-198						
24.	$-\frac{1}{2}$						
25.	10						

ANSWERS

1. IN, R **3.** RN, R **5.** RN, R **7.** IN, R **9.** None **11.** RN, R **13.** RN, R **15.** I, RN, R
17. I, RN, R **19.** RN, R **21.** RN, R **23.** I, RN, R **25.** N, W, I, RN, R

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Operations with Integers

Multiplying and Dividing Integers

Rules for Multiplying and Dividing Integers	
Multiplying and Dividing:	The product or quotient of two integers with the *same* sign is *positive*. The product or quotient of two integers with *different* signs is *negative*.

Example 5 **Find (a) $-7 \cdot (-1)$ and (b) $-9 \cdot 4$.**

a. $-7 \cdot (-1) = 7$ The integers have the same sign, so the product is positive.

▶ The product is 7.

b. $-9 \cdot 4 = -36$ The integers have different signs, so the product is negative.

▶ The product is -36.

Example 6 **Find (a) $18 \div (-2)$ and (b) $-25 \div (-5)$.**

a. $18 \div (-2) = -9$ The integers have different signs, so the quotient is negative.

▶ The quotient is -9.

b. $-25 \div (-5) = 5$ The integers have the same sign, so the quotient is positive.

▶ The quotient is 5.

Practice

Check your answers at BigIdeasMath.com.

Find the absolute value.

1. $|13|$ **2.** $|-8|$ **3.** $|0|$ **4.** $|-297|$

Evaluate.

5. $5 + (-11)$ **6.** $4 - 9$ **7.** $-15 + (-10)$ **8.** $9 + (-6)$

9. $0 - (-50)$ **10.** $-8 + 20$ **11.** $-11 - 11$ **12.** $-14 + 0$

13. $20 - (-21)$ **14.** $-34 - (-25)$ **15.** $-8 + (-3) + 6$ **16.** $1 + 7 - 9$

Simplify the expression.

17. $|-15 - 9|$ **18.** $|18 - (-11)|$ **19.** $|-14 + 17|$ **20.** $|-24 - (-19)|$

Evaluate.

21. $-8 \cdot 25$ **22.** $-33 \div (-3)$ **23.** $-13(-1)$ **24.** $-24 \div 4$

25. $0(-4)$ **26.** $-15(8)$ **27.** $\frac{0}{-12}$ **28.** $-1(-1)$

29. $\frac{-16}{-1}$ **30.** $240 \div (-8)$ **31.** $5 \cdot (-7) \cdot (-4)$ **32.** $12 \div (-3) \cdot 2$

33. **ELEVATION** The highest elevation in California is 14,494 feet, on Mount Whitney. The lowest elevation in California is -282 feet in Death Valley. Find the range of elevations in California.

34. **GOLF** The table shows a golfer's score for each round of a tournament. Find the golfer's total score and the golfer's mean score per round.

	Round 1	Round 2	Round 3
Score	-3	-4	$+1$

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Order of Operations

To evaluate numerical expressions, use a set of rules called the **order of operations.**

Order of Operations
1. Perform operations in Parentheses.
2. Evaluate numbers with Exponents.
3. Multiply or Divide from left to right.
4. Add or Subtract from left to right.

Example 1 **Evaluate each expression.**

a. $20 - 5 \cdot 6$

$20 - 5 \cdot 6 = 20 - 30$ — Multiply 5 and 6.

$= -10$ — Subtract 30 from 20.

b. $12 \cdot 3 + 4^2 \div 8$

$12 \cdot 3 + 4^2 \div 8 = 12 \cdot 3 + 16 \div 8$ — Evaluate 4^2.

$= 36 + 16 \div 8$ — Multiply 12 and 3.

$= 36 + 2$ — Divide 16 by 8.

$= 38$ — Add 36 and 2.

c. $7(5 - 3) + 6^2 \div (-3)$

$7(5 - 3) + 6^2 \div (-3) = 7(2) + 6^2 \div (-3)$ — Perform operation in parentheses.

$= 7(2) + 36 \div (-3)$ — Evaluate 6^2.

$= 14 + 36 \div (-3)$ — Multiply 7 and 2.

$= 14 + (-12)$ — Divide 36 by −3.

$= 2$ — Add 14 and −12.

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression.

1. $8 + 2 \cdot 5$

2. $40 \div 8 - 7$

3. $5 \cdot 4^2 \div 8$

4. $1 - 7 + 5^2$

5. $\dfrac{3 - (-9)}{-10 + 6}$

6. $\dfrac{2 + 4}{1 - 5} - 1$

7. $(12 - 8)^2 \div 2^5$

8. $18 + 9^2 - 7 \cdot (-3)$

9. $32 \div 8 + 2 \cdot 8^2$

10. $6 \div (7 \div 28)$

11. $36 \div \left(1 - |2 - 7|\right)$

12. $(-2)^2 \cdot 5 - 7(9 - 5)$

13. $4(3 + 8) - 8^2 \div 32$

14. $10(3 - 6)^3 + 41$

15. $(2 - 5)^2 - (4 \cdot 5^2)$

16. **RESTAURANT** There are 82 people in a restaurant. Four groups of 3 leave and then five groups of 2 enter. Evaluate the expression $82 - 4(3) + 5(2)$ to find how many people are in the restaurant.

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Powers and Exponents

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base → $\left(\frac{2}{5}\right)^3$ ← exponent

$$\underbrace{\left(\frac{2}{5}\right)^3}_{\text{power}} = \underbrace{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}}_{\frac{2}{5}\text{ is used as factor 3 times}}$$

Example 1 Write each product using exponents.

a. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

Because -9 is used as a factor 5 times, its exponent is 5.

▶ So, $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^5$.

b. $\pi \cdot \pi \cdot h \cdot h \cdot h$

Because π is used as a factor 2 times, its exponent is 2. Because h is used as a factor 3 times, its exponent is 3.

▶ So, $\pi \cdot \pi \cdot h \cdot h \cdot h = \pi^2 h^3$.

Example 2 Evaluate each expression.

a. $(-5)^4$

$(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5)$ Write as repeated multiplication.

$= 625$ Simplify.

b. -5^4

$-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5)$ Write as repeated multiplication.

$= -625$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

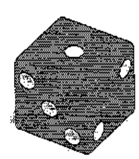
Write the product using exponents.

1. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$
2. $\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right)$
3. $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$
4. $2.5 \cdot 2.5 \cdot b \cdot b \cdot b \cdot b$
5. $(-n) \cdot (-n) \cdot (-n) \cdot (-n)$
6. $(-12) \cdot (-12) \cdot v \cdot v \cdot v$

Evaluate the expression.

7. 10^4
8. -15^2
9. $\left(\frac{3}{4}\right)^3$
10. $\left(-\frac{1}{2}\right)^5$

11. **VOLUME** Write an expression involving a power that represents the volume (in cubic centimeters) of the die shown. Then find the volume.



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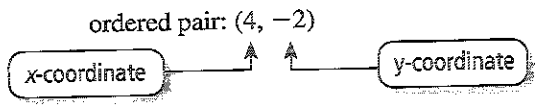
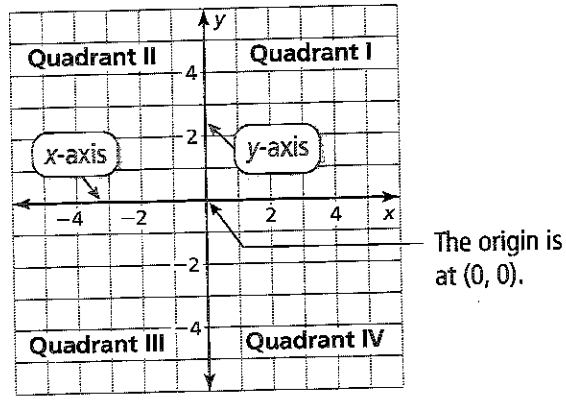
The Coordinate Plane

A **coordinate plane** is formed by the intersection of a horizontal number line and a vertical number line. The number lines intersect at the **origin** and separate the coordinate plane into four regions called **quadrants**.

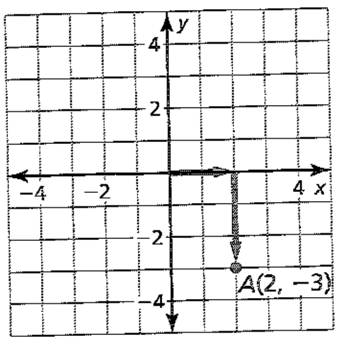
An **ordered pair** is used to locate a point in a coordinate plane.



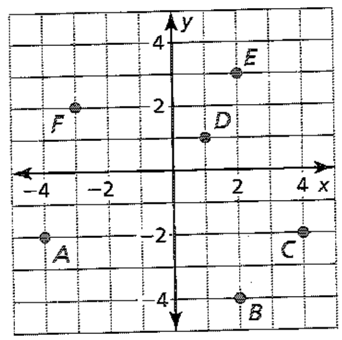


Example 1 Plot the point $A(2, -3)$ in a coordinate plane. Describe the location of the point.

Start at the origin. Move 2 units right and 3 units down. Then plot the point. The point is in Quadrant IV.



Example 2 What ordered pair corresponds to point *A*?



Point A is 4 units to the left of the origin and 2 units down. So, the x-coordinate is -4 and the y-coordinate is -2.

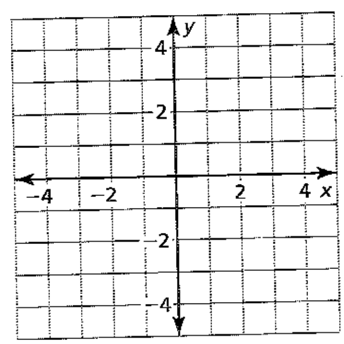
▶ The ordered pair $(-4, -2)$ corresponds to point A.

Practice

Check your answers at BigIdeasMath.com.

Plot the ordered pair in a coordinate plane. Describe the location of the point.

1. $A(1, 3)$
2. $B(-2, 2)$
3. $C(2, -4)$
4. $D(1, -1)$
5. $E(-4, -2.5)$
6. $F(-3, 0)$
7. $G(0, 1)$
8. $H\left(4, \frac{1}{2}\right)$



Use the graph in Example 2 to answer the questions.

9. What ordered pair corresponds to point C?
10. What ordered pair corresponds to point F?

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The Distributive Property

To multiply a sum or difference by a number, multiply each number in the sum or difference by the number outside the parentheses, then evaluate.

Distributive Property	
With addition: $5(7 + 3) = 5(7) + 5(3)$	$a(b + c) = a(b) + a(c)$
With subtraction: $5(7 - 3) = 5(7) - 5(3)$	$a(b - c) = a(b) - a(c)$

Example 2 **Simplify each expression.**

a. $6(x + 9)$

$6(x + 9) = 6(x) + 6(9)$

$= 6x + 54$

b. $10(12 + z + 7)$

$10(12 + z + 7) = 10(12) + 10(z) + 10(7)$

$= 120 + 10z + 70$

$= 10z + 190$

c. $16(8w - 3)$

$16(8w - 3) = 16(8w) - 16(3)$

$= 128w - 48$

d. $5(4m - 3n - 1)$

$5(4m - 3n - 1) = 5(4m) - 5(3n) - 5(1)$

$= 20m - 15n - 5$

Practice

Check your answers at BigIdeasMath.com.

Evaluate.

1. $25(7 + 11)$ **2.** $4(13 - 5)$ **3.** $9(16 + 7 - 8)$ **4.** $-4(10 - 9 - 6)$

Simplify the expression.

5. $4(y + 7)$ **6.** $-2(z + 5)$ **7.** $5(b - 11)$ **8.** $-8(d - 1)$

9. $12(4a + 13)$ **10.** $9(20 + 17m)$ **11.** $11(2k - 11)$ **12.** $-7(-2n - 9)$

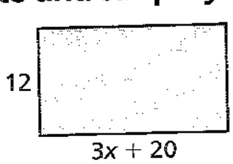
13. $3(x + 4 + 9)$ **14.** $6(25 + 6z + 10)$ **15.** $8(p - 6 - 5)$ **16.** $-10(4 + v - 1)$

17. $7(2x + 7 + 9y)$ **18.** $-4(4r - s + 17)$ **19.** $-3(-12 - 3d - 8)$ **20.** $2 - 6(2n - 9)$

21. $1.5(6c + 10d + 3)$ **22.** $\frac{3}{4}\left(q + \frac{1}{6} + \frac{7}{8}\right)$ **23.** $-2.4(5h - 10 + 4)$ **24.** $0.5(2.6x + 5.8)$

Write and simplify an expression for the area of the rectangle.

7.



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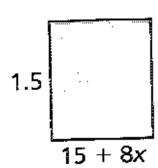

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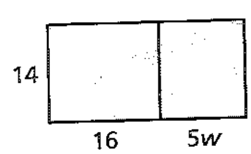

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Simplifying Algebraic Expressions

Parts of an algebraic expression are called *terms*. **Like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.

An algebraic expression is in **simplest form** when it has no like terms and no parentheses. To *combine* like terms that have variables, use the Distributive Property to add or subtract the coefficients.

Example 1 **Simplify $8y + 7y$.**

$8y + 7y = (8 + 7)y$	Distributive Property
$= 15y$	Add coefficients.

Example 2 **Simplify $2(x + 5) - 3(x - 2)$.**

$2(x + 5) - 3(x - 2) = 2(x) + 2(5) - 3(x) - 3(-2)$	Distributive Property
$= 2x + 10 - 3x + 6$	Multiply.
$= 2x - 3x + 10 + 6$	Group like terms.
$= -x + 16$	Combine like terms.

Example 3 **Simplify $xy + 3y - 2x + 5y - 3xy$.**

$xy + 3y - 2x + 5y - 3xy = xy - 3xy + 3y + 5y - 2x$	Group like terms.
$= -2xy + 8y - 2x$	Combine like terms.

Practice

Check your answers at BigIdeasMath.com.

Simplify the expression.

1. $7x + 15x$

2. $8y - 14y$

3. $7d + 9 - 5d$

4. $3w + 2(2 - 3w) + 2$

5. $(x + 3) + (3x - 7)$

6. $(5k + 6) + (4k - 8)$

7. $(-7n + 6) + (5n + 15)$

8. $(9z + 12) - (6z + 8)$

9. $(8b + 1) - (-10b - 5)$

10. $s(8 - 2t) + 3t(4 - 2s) + 5t$

11. $qr + 2q^2 - 3qr - r^2 - 6q^2$

12. $g^3(h - 4g) - h(3 - 2g^3)$

13. **EARNINGS** The original price of a model car is d dollars. You use a coupon and buy the kit for $(d - 10)$ dollars. You assemble the model car and sell it for $(2d - 20)$ dollars. Write an expression that represents your earnings. Interpret the expression.

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Solving Linear Equations

To determine whether a value is a solution of an equation, substitute the value into the equation and simplify.

Example 1 **Determine whether (a) $x = 1$ or (b) $x = -2$ is a solution of $5x - 1 = 4$.**

a. $5x - 1 = -2x + 6$

$5(1) - 1 \stackrel{?}{=} -2(1) + 6$ Substitute.

$4 = 4$ ✓ Simplify.

▶ So, $x = 1$ is a solution.

b. $5x - 1 = -2x + 6$

$5(-2) - 1 \stackrel{?}{=} -2(-2) + 6$ Substitute.

$-11 \neq 10$ ✗ Simplify.

▶ So, $x = -2$ is *not* a solution.

To solve a linear equation, isolate the variable.

Example 2 **Solve each equation. Check your solution.**

a. $4x - 3 = 13$

$4x - 3 + 3 = 13 + 3$ Add 3.

$4x = 16$ Simplify.

$\frac{4x}{4} = \frac{16}{4}$ Divide by 4.

$x = 4$ Simplify.

Check

$4x - 3 = 13$

$4(4) - 3 \stackrel{?}{=} 13$

$13 = 13$ ✓

b. $2(y - 8) = y + 6$

$2y - 16 = y + 6$ Distributive Property

$2y - y - 16 = y - y + 6$ Subtract y.

$y - 16 = 6$ Simplify.

$y - 16 + 16 = 6 + 16$ Add 16.

$y = 22$ Simplify.

Check

$2(y - 8) = y + 6$

$2(22 - 8) \stackrel{?}{=} 22 + 6$

$28 = 28$ ✓

Practice

Check your answers at BigIdeasMath.com.

Determine whether (a) $x = -1$ or (b) $x = 3$ is a solution of the equation.

1. $5x + 7 = 2$
2. $-4x + 8 = -4$
3. $2x - 1 = 3x - 4$

Solve the equation. Check your solution.

4. $x - 9 = 24$
5. $n + 14 = 0$
6. $-16 = 4y$
7. $-\frac{5}{6}t = -15$
8. $81 = 46 - x$
9. $4x + 5 = 1$
10. $x + 5 = 11x$
11. $9(y - 3) = 45$
12. $6 = 7k + 8 - k$
13. $6n + 3 = -4n + 7$
14. $2c + 5 = 3(c - 8)$
15. $18m + 3(2m + 8) = 0$
16. $\frac{w - 6}{5} = 8$
17. $\frac{15 + h}{3} = 10$
18. $\frac{8 - 3x}{5} = x$
19. $(8r + 6) + (4r - 1) = 14$
20. $\frac{2}{3}y - 3 = 9$
21. $\frac{1}{2}x - \frac{3}{10} = \frac{5}{2}x + \frac{7}{10}$

22. **MONEY** You have a total of \$3.25 in change made up of 25 pennies, 6 nickels, 2 dimes, and x quarters. How many quarters do you have?

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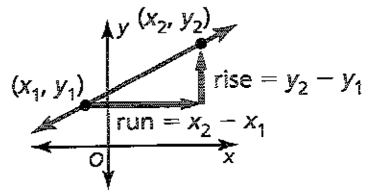
Slope of a Line

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line. If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2), then the slope m is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$



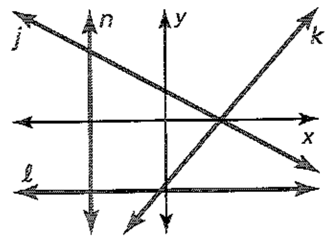
Slopes of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line *j*

Positive slope: rises from left to right, as in line *k*

Zero slope (slope of 0): horizontal, as in line *ℓ*

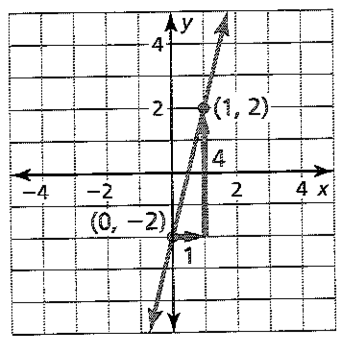
Undefined slope: vertical, as in line *n*



Example 1 Find the slope of the line shown.

Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (1, 2)$.

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$	Write formula for slope.
$= \frac{2 - (-2)}{1 - 0}$	Substitute.
$= 4$	Simplify.



Practice

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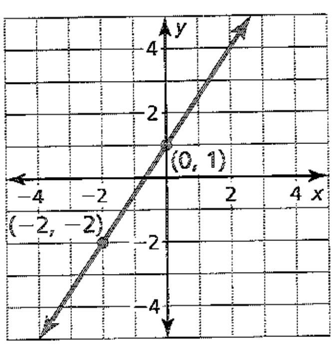
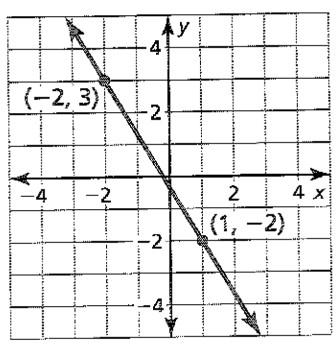
Find the slope of the line.

1.



2.



3.