Name _____ Date _____

Prime Factorization

Because 3 is a factor of 24 and $3 \cdot 8 = 24$, 8 is also a factor of 24. The pair 3, 8 is called a **factor pair** of 24.

The **prime factorization** of a composite number is the number written as a product of its prime factors. You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product.

Example 1 **A classroom has 42 students. The teacher arranges the students in rows. Each row has the same number of students. How many possible arrangements are there?**

Use the factor pairs of 42 to find the number of arrangements.

$42 = 1 \cdot 42$ 1 row of 42 or 42 rows of 1 $42 = 2 \cdot 21$ 2 rows of 21 or 21 rows of 2

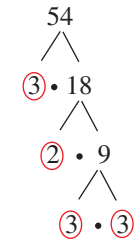
$42 = 3 \cdot 14$ 3 rows of 14 or 14 rows of 3 $42 = 6 \cdot 7$ 6 rows of 7 or 7 rows of 6

▶ There are 8 possible arrangements: 1 row of 42, 42 rows of 1, 2 rows of 21, 21 rows of 2, 3 rows of 14, 14 rows of 3, 6 rows of 7, or 7 rows of 6.

Example 2 **Write the prime factorization of 54.**

Choose any factor pair of 54 to begin the factor tree.

Tree 1



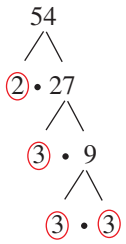
Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

$54 = 3 \cdot 2 \cdot 3 \cdot 3$

Tree 2



$54 = 2 \cdot 3 \cdot 3 \cdot 3$

▶ The prime factorization of 54 is $2 \cdot 3 \cdot 3 \cdot 3$, or $2 \cdot 3^3$.

Practice

Check your answers at BigIdeasMath.com.

List the factor pairs of the number.

1. 16 **2.** 30 **3.** 63

4. 100 **5.** 135 **6.** 275

Write the prime factorization of the number.

7. 24 **8.** 66 **9.** 50 **10.** 80

11. 98 **12.** 126 **13.** 154 **14.** 310

Find the greatest perfect square that is a factor of the number.

15. 117 **16.** 150 **17.** 539 **18.** 936

19. **EXERCISE** An exercise class has 28 participants. The instructor arranges the participants in rows. Each row has the same number of participants. How many possible arrangements are there?

Name _____ Date _____

Greatest Common Factor

Factors that are shared by two or more numbers are called **common factors.** The greatest of the common factors is called the **greatest common factor** (GCF). There are several different ways to find the GCF of two or more numbers.

Example 1 **Find the greatest common factor (GCF) of 56 and 104.**

Method 1 List the factors of each number. Then circle the common factors.

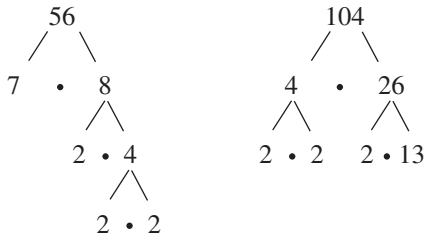
Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56

Factors of 104: 1, 2, 4, 8, 13, 26, 52, 104

The common factors are 1, 2, 4, and 8. The greatest of these common factors is 8.

▶ So, the GCF of 56 and 104 is 8.

Method 2 Make a factor tree for each number.



Write the prime factorization of each number. Then circle the common prime factors. The GCF is the product of the common prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$

$104 = 2 \cdot 2 \cdot 2 \cdot 13$

▶ So, the GCF of 56 and 104 is $2 \cdot 2 \cdot 2 = 8$.

Practice

Check your answers at BigIdeasMath.com.

Find the GCF of the numbers using the two methods shown above.

1. 30, 45

2. 12, 54

3. 16, 96

4. 42, 98

5. 27, 66

6. 50, 160

7. 21, 70

8. 76, 95

9. 60, 84

10. 60, 120, 210

11. 44. 64, 100

12. 15, 28, 70

13. Write a set of two numbers that have a GCF of 20. Explain how you found your answer.

14. Write a set of three numbers that have a GCF of 25. Explain how you found your answer.

15. **BOUQUETS** A florist is making identical bouquets using 90 white roses, 60 red roses, and 45 pink roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?

16. **FABRIC** You have two pieces of fabric. One piece is 6 feet wide and the other piece is 7.5 feet wide. You want to cut both pieces into strips of equal width that are as wide as possible. How wide should you cut the strips of fabric?

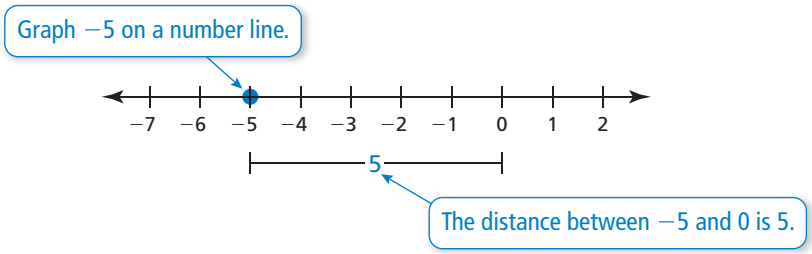
Name _____ Date _____

Operations with Integers

Adding and Subtracting Integers

The **absolute value** of an integer is the distance between the number and 0 on a number line. The absolute value of a number x is written as $|x|$.

Example 1 **Find the absolute value of -5.**



▶ So, $|-5| = 5$.

Rules for Adding and Subtracting Integers	
Adding:	To add integers with the *same* sign, add the absolute values of the integers. Then use the common sign. To add integers with *different* signs, subtract the lesser absolute value from the greater absolute value. Then use the sign of the integer with the greater absolute value.
Subtracting:	To subtract an integer, add its opposite.

Example 2 **Find (a) $-3 + (-8)$ and (b) $-9 + 6$.**

a. $-3 + (-8) = -11$ Add $|-3|$ and $|-8|$. Use the common sign.

▶ The sum is -11.

b. $-9 + 6 = -3$ $|-9| > |6|$. So, subtract $|6|$ from $|-9|$. Use the sign of -9.

▶ The sum is -3.

Example 3 **Find (a) $5 - (-12)$ and (b) $1 - 7$.**

a. $5 - (-12) = 5 + 12$ Add the opposite of -12.

$= 17$ Add.

▶ The difference is 17.

b. $1 - 7 = 1 + (-7)$ Add the opposite of 7.

$= -6$ Add.

▶ The difference is -6.

Example 4 **Simplify $|-14 - (-10)|$.**

$-14 - (-10) = |-14 + 10|$ Add the opposite of -10.

$= |-4|$ Add.

$= 4$ Find the absolute value.

▶ So, $|-14 - (-10)| = 4$.

Name _____ Date _____

Operations with Integers

Multiplying and Dividing Integers

Rules for Multiplying and Dividing Integers
Multiplying and Dividing: The product or quotient of two integers with the *same* sign is *positive*. The product or quotient of two integers with *different* signs is *negative*.

Example 5 **Find (a) $-7 \cdot (-1)$ and (b) $-9 \cdot 4$.**

a. $-7 \cdot (-1) = 7$ The integers have the same sign, so the product is positive.

▶ The product is 7.

b. $-9 \cdot 4 = -36$ The integers have different signs, so the product is negative.

▶ The product is -36.

Example 6 **Find (a) $18 \div (-2)$ and (b) $-25 \div (-5)$.**

a. $18 \div (-2) = -9$ The integers have different signs, so the quotient is negative.

▶ The quotient is -9.

b. $-25 \div (-5) = 5$ The integers have the same sign, so the quotient is positive.

▶ The quotient is 5.

Practice

Check your answers at BigIdeasMath.com.

Find the absolute value.

1. $|13|$ **2.** $|-8|$ **3.** $|0|$ **4.** $|-297|$

Evaluate.

5. $5 + (-11)$ **6.** $4 - 9$ **7.** $-15 + (-10)$ **8.** $9 + (-6)$

9. $0 - (-50)$ **10.** $-8 + 20$ **11.** $-11 - 11$ **12.** $-14 + 0$

13. $20 - (-21)$ **14.** $-34 - (-25)$ **15.** $-8 + (-3) + 6$ **16.** $1 + 7 - 9$

Simplify the expression.

17. $|-15 - 9|$ **18.** $|18 - (-11)|$ **19.** $|-14 + 17|$ **20.** $|-24 - (-19)|$

Evaluate.

21. $-8 \cdot 25$ **22.** $-33 \div (-3)$ **23.** $-13(-1)$ **24.** $-24 \div 4$

25. $0(-4)$ **26.** $-15(8)$ **27.** $\frac{0}{-12}$ **28.** $-1(-1)$

29. $\frac{-16}{-1}$ **30.** $240 \div (-8)$ **31.** $5 \cdot (-7) \cdot (-4)$ **32.** $12 \div (-3) \cdot 2$

33. **ELEVATION** The highest elevation in California is 14,494 feet, on Mount Whitney. The lowest elevation in California is -282 feet in Death Valley. Find the range of elevations in California.

34. **GOLF** The table shows a golfer's score for each round of a tournament. Find the golfer's total score and the golfer's mean score per round.

	Round 1	Round 2	Round 3
Score	-3	-4	$+1$

Name _____ Date _____

Evaluating Algebraic Expressions

An **algebraic expression** is an expression that may contain numbers, operations, and one or more symbols. A symbol that represents one or more numbers is called a **variable**. To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

Example 1 **Evaluate each expression when $x = 3$.**

a. $5x + 7$

$5x + 7 = 5(3) + 7$ Substitute 3 for x.

$= 15 + 7$ Multiply.

$= 22$ Add.

b. $14 - x^2$

$14 - x^2 = 14 - 3^2$ Substitute 3 for x.

$= 14 - 9$ Evaluate power.

$= 5$ Subtract.

c. $2x^2 - 8x + 4$

$2x^2 - 8x + 4 = 2(3)^2 - 8(3) + 4$ Substitute 3 for x.

$= 2(9) - 8(3) + 4$ Evaluate power.

$= 18 - 24 + 4$ Multiply.

$= -2$ Simplify.

Example 2 **Evaluate each expression when $x = -2$ and $y = 6$.**

a. $7x - 5y$

$7x - 5y = 7(-2) - 5(6)$

$= -14 - 30$

$= -44$

b. $x^2 - 2xy + y^2$

$x^2 - 2xy + y^2 = (-2)^2 - 2(-2)(6) + 6^2$

$= 4 - 2(-2)(6) + 36$

$= 4 - (-24) + 36$

$= 64$

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression when $x = 2$ and $y = -3$.

1. $3x + 10$

2. $14 - 2y$

3. $5 - y^2$

4. $4x^2 + 9$

5. $y^2 + 8y - 4$

6. $-3x^2 - x + 7$

7. $0.75x - 4x - 1.5$

8. $3(y + 8 - 4y)$

9. $2x + 3y$

10. $6y - 5x$

11. $4x^2 + 3y$

12. $x^2 - y^2$

13. $y - x + y^2$

14. $x^2y^2 + xy$

15. $\frac{x + y}{y - x}$

16. $\frac{2x + y}{xy}$

Copy and complete the table.

17.

x	0	1	2	3	4
$3x - 2$					

18.

x	-2	-1	0	1	2
$-4x + 1$					

19. **MONEY** You earn $8x + 7y$ dollars for working x hours at a restaurant and y hours at a bus station. How much do you earn for working 12 hours at the restaurant and 16 hours at the bus station?

Name _____ Date _____

The Distributive Property

To multiply a sum or difference by a number, multiply each number in the sum or difference by the number outside the parentheses, then evaluate.

Distributive Property	
With addition: $5(7 + 3) = 5(7) + 5(3)$	$a(b + c) = a(b) + a(c)$
With subtraction: $5(7 - 3) = 5(7) - 5(3)$	$a(b - c) = a(b) - a(c)$

Example 2 **Simplify each expression.**

a. $6(x + 9)$

$6(x + 9) = 6(x) + 6(9)$

$= 6x + 54$

b. $10(12 + z + 7)$

$10(12 + z + 7) = 10(12) + 10(z) + 10(7)$

$= 120 + 10z + 70$

$= 10z + 190$

c. $16(8w - 3)$

$16(8w - 3) = 16(8w) - 16(3)$

$= 128w - 48$

d. $5(4m - 3n - 1)$

$5(4m - 3n - 1) = 5(4m) - 5(3n) - 5(1)$

$= 20m - 15n - 5$

Practice

Check your answers at BigIdeasMath.com.

Evaluate.

1. $25(7 + 11)$ **2.** $4(13 - 5)$ **3.** $9(16 + 7 - 8)$ **4.** $-4(10 - 9 - 6)$

Simplify the expression.

5. $4(y + 7)$ **6.** $-2(z + 5)$ **7.** $5(b - 11)$ **8.** $-8(d - 1)$

9. $12(4a + 13)$ **10.** $9(20 + 17m)$ **11.** $11(2k - 11)$ **12.** $-7(-2n - 9)$

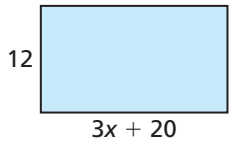
13. $3(x + 4 + 9)$ **14.** $6(25 + 6z + 10)$ **15.** $8(p - 6 - 5)$ **16.** $-10(4 + v - 1)$

17. $7(2x + 7 + 9y)$ **18.** $-4(4r - s + 17)$ **19.** $-3(-12 - 3d - 8)$ **20.** $2 - 6(2n - 9)$

21. $1.5(6c + 10d + 3)$ **22.** $\frac{3}{4}\left(q + \frac{1}{6} + \frac{7}{8}\right)$ **23.** $-2.4(5h - 10 + 4)$ **24.** $0.5(2.6x + 5.8)$

Write and simplify an expression for the area of the rectangle.

7.



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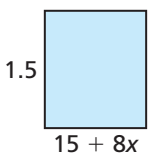

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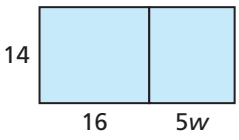

Name _____ Date _____

Order of Operations

To evaluate numerical expressions, use a set of rules called the **order of operations**.

Order of Operations
1. Perform operations in **P**arentheses.
2. Evaluate numbers with **E**xponents.
3. **M**ultiply or **D**ivide from left to right.
4. **A**dd or **S**ubtract from left to right.

Example 1 **Evaluate each expression.**

a. $20 - 5 \cdot 6$

$20 - 5 \cdot 6 = 20 - 30$ Multiply 5 and 6.

$= -10$ Subtract 30 from 20.

b. $12 \cdot 3 + 4^2 \div 8$

$12 \cdot 3 + 4^2 \div 8 = 12 \cdot 3 + 16 \div 8$ Evaluate 4^2.

$= 36 + 16 \div 8$ Multiply 12 and 3.

$= 36 + 2$ Divide 16 by 8.

$= 38$ Add 36 and 2.

c. $7(5 - 3) + 6^2 \div (-3)$

$7(5 - 3) + 6^2 \div (-3) = 7(2) + 6^2 \div (-3)$ Perform operation in parentheses.

$= 7(2) + 36 \div (-3)$ Evaluate 6^2.

$= 14 + 36 \div (-3)$ Multiply 7 and 2.

$= 14 + (-12)$ Divide 36 by -3.

$= 2$ Add 14 and -12.

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression.

1. $8 + 2 \cdot 5$

2. $40 \div 8 - 7$

3. $5 \cdot 4^2 \div 8$

4. $1 - 7 + 5^2$

5. $\dfrac{3 - (-9)}{-10 + 6}$

6. $\dfrac{2 + 4}{1 - 5} - 1$

7. $(12 - 8)^2 \div 2^5$

8. $18 + 9^2 - 7 \cdot (-3)$

9. $32 \div 8 + 2 \cdot 8^2$

10. $6 \div (7 \div 28)$

11. $36 \div \left(1 - |2 - 7|\right)$

12. $(-2)^2 \cdot 5 - 7(9 - 5)$

13. $4(3 + 8) - 8^2 \div 32$

14. $10(3 - 6)^3 + 41$

15. $(2 - 5)^2 - (4 \cdot 5^2)$

16. **RESTAURANT** There are 82 people in a restaurant. Four groups of 3 leave and then five groups of 2 enter. Evaluate the expression $82 - 4(3) + 5(2)$ to find how many people are in the restaurant.

Name _____ Date _____

Solving Linear Equations

To determine whether a value is a solution of an equation, substitute the value into the equation and simplify.

Example 1 **Determine whether (a) $x = 1$ or (b) $x = -2$ is a solution of $5x - 1 = 4$.**

a. $5x - 1 = -2x + 6$

$5(1) - 1 \stackrel{?}{=} -2(1) + 6$ Substitute.

$4 = 4$ ✓ Simplify.

▶ So, $x = 1$ is a solution.

b. $5x - 1 = -2x + 6$

$5(-2) - 1 \stackrel{?}{=} -2(-2) + 6$ Substitute.

$-11 \neq 10$ ✗ Simplify.

▶ So, $x = -2$ is *not* a solution.

To solve a linear equation, isolate the variable.

Example 2 **Solve each equation. Check your solution.**

a. $4x - 3 = 13$

$4x - 3 + 3 = 13 + 3$ Add 3.

$4x = 16$ Simplify.

$\frac{4x}{4} = \frac{16}{4}$ Divide by 4.

$x = 4$ Simplify.

Check

$4x - 3 = 13$

$4(4) - 3 \stackrel{?}{=} 13$

$13 = 13$ ✓

b. $2(y - 8) = y + 6$

$2y - 16 = y + 6$ Distributive Property

$2y - y - 16 = y - y + 6$ Subtract y.

$y - 16 = 6$ Simplify.

$y - 16 + 16 = 6 + 16$ Add 16.

$y = 22$ Simplify.

Check

$2(y - 8) = y + 6$

$2(22 - 8) \stackrel{?}{=} 22 + 6$

$28 = 28$ ✓

Practice

Check your answers at BigIdeasMath.com.

Determine whether (a) $x = -1$ or (b) $x = 3$ is a solution of the equation.

1. $5x + 7 = 2$
2. $-4x + 8 = -4$
3. $2x - 1 = 3x - 4$

Solve the equation. Check your solution.

4. $x - 9 = 24$
5. $n + 14 = 0$
6. $-16 = 4y$
7. $-\frac{5}{6}t = -15$
8. $81 = 46 - x$
9. $4x + 5 = 1$
10. $x + 5 = 11x$
11. $9(y - 3) = 45$
12. $6 = 7k + 8 - k$
13. $6n + 3 = -4n + 7$
14. $2c + 5 = 3(c - 8)$
15. $18m + 3(2m + 8) = 0$
16. $\frac{w - 6}{5} = 8$
17. $\frac{15 + h}{3} = 10$
18. $\frac{8 - 3x}{5} = x$
19. $(8r + 6) + (4r - 1) = 14$
20. $\frac{2}{3}y - 3 = 9$
21. $\frac{1}{2}x - \frac{3}{10} = \frac{5}{2}x + \frac{7}{10}$

22. **MONEY** You have a total of \$3.25 in change made up of 25 pennies, 6 nickels, 2 dimes, and x quarters. How many quarters do you have?

Name _____ Date _____

Solving Proportions

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. To solve proportions, use the Cross Products Property.

Cross Products Property	
Words The cross products of a proportion are equal.	
Numbers	**Algebra**
$\frac{2}{3} = \frac{4}{6}$	$\frac{a}{b} = \frac{c}{d}$
$2 \cdot 6 = 3 \cdot 4$	$ad = bc$, where $b \neq 0$ and $d \neq 0$

Example 1 **Solve each proportion.**

a. $\frac{x}{6} = \frac{5}{2}$

$x \cdot 2 = 6 \cdot 5$ Cross Products Property

$2x = 30$ Multiply.

$x = 15$ Divide.

▶ The solution is 15.

b. $\frac{8}{y} = \frac{4}{9}$

$8 \cdot 9 = y \cdot 4$

$72 = 4y$

$18 = y$

▶ The solution is 18.

Practice

Check your answers at BigIdeasMath.com.

Solve the proportion.

1. $\frac{1}{3} = \frac{x}{6}$ **2.** $\frac{2}{5} = \frac{y}{10}$ **3.** $\frac{z}{9} = \frac{2}{3}$ **4.** $\frac{2}{7} = \frac{j}{14}$

5. $\frac{4}{9} = \frac{k}{36}$ **6.** $\frac{m}{24} = \frac{3}{8}$ **7.** $\frac{11}{3} = \frac{p}{6}$ **8.** $\frac{n}{54} = \frac{8}{3}$

9. $\frac{14}{a} = \frac{7}{2}$ **10.** $\frac{15}{b} = \frac{3}{5}$ **11.** $\frac{21}{2} = \frac{42}{d}$ **12.** $\frac{9}{16} = \frac{27}{g}$

13. $\frac{21}{r} = \frac{7}{5}$ **14.** $\frac{25}{q} = \frac{5}{2}$ **15.** $\frac{9}{8} = \frac{36}{s}$ **16.** $\frac{4}{15} = \frac{20}{t}$

17. $\frac{x}{2.4} = \frac{3.1}{1.2}$ **18.** $\frac{4.8}{1.5} = \frac{m}{4.5}$ **19.** $\frac{3.3}{y} = \frac{1.1}{1.6}$ **20.** $\frac{2.8}{5.4} = \frac{1.4}{c}$

21. PENCILS Thirty-six pencils are packaged in 6 boxes. How many pencils are packaged in 10 boxes?

22. TICKETS Two tickets cost $15. How much does it cost to buy seven tickets?

23. SALADS Three salads cost $6.50. How much does it cost to buy six salads?

24. FIELD TRIP There are 108 students on a field trip. The ratio of girls to boys is 5 to 4. How many are girls?